Conclusions

Analyses for the supersonic flutter of a system of elastically connected parallel plates have been made and general expressions derived for the appropriate nondimensional parameters as a function of geometry, stiffnesses, and inplane loadings. The influence of the elastic connecting medium is critical in determining the severity of the flutter and it is also found that flutter may be more critical for large tensile values of the midplane (chordwise) stress resultant. The finite difference formulation gives most encouraging results from quite simple analyses and deserves further development particularly for panels with irregular planform contours.

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Nonlinear Equations for Shallow Sandwich Shells with Orthotropic Cores

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Nomenclature

B_i = $E_i t_i/(1-\mu^2)$
C = thickness of the core
C_1 = $(1-\mu^2)\ (B_1+B_2)$
D_i = $E_i t_i^3/12(1-\mu^2)$
E_i = elastic modulus of the ith facing
G_{xz}, G_{yz} = shear moduli of the core in the x and y directions, respectively
h = $C+(t_1+t_2)/2$
k = $CB_1B_2/G_{xz}(B_1+B_2)$
k_1 = G_{xz}/G_{yz}
$\bar{k}$ = $h^2B_1B_2/(B_1+B_2)$
N_u^*, N_L^* = upper and lower critical loads
R_1, R_2 = principal radii of curvature
t_i = thickness of the ith facing
w = deflection of sandwich shell
w_0 = undetermined coefficient

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I. Introduction

NONLINEAR equations for a shallow unsymmetrical sandwich shell of double curvature were obtained by Fulton.[1] However, in his analysis he treated the core to be isotropic, while in the actual constructions the cores are usually orthotropic. The purpose of this Note is to make a generalization of Fulton's equations[1] by including the effect of core orthotropy. The assumptions made in this Note with regard to faces and core are the same as those in Fulton's paper except the core is considered as orthotropic in this Note. As an example a square cylindrical shell section loaded in the longitudinal direction is considered, and the effects of core orthotropy upon the lower and upper critical loads are discussed.

II. Governing Equations

The same analytical procedure used in Ref. 1 can be used in this derivation. However, because of the presence of orthotropic core, the derivation becomes much more complex. After a rather lengthy process of substituion and differentiation, the following nonlinear eqations for shallow unsymmetrical sandwich shell of double curvature with orthotropic core are obtained[2]:

$$\nabla^4 F+(1-\mu^2)\ (B_1+B_2)\left[\frac{1}{R_1}\frac{\partial^2 w}{\partial y^2}+\frac{1}{R_2}\frac{\partial^2 w}{\partial x^2}+\frac{\partial^2 w}{\partial x^2}\frac{\partial^2 w}{\partial y^2}-\left[\frac{\partial^2 w}{\partial x \partial y}\right]^2\right]=0 \tag{1}$$

$$\left[k\left[\frac{\partial^2}{\partial y^2}+k_1\frac{\partial^2}{\partial x^2}\right]-\frac{2}{(1-\mu)}\right] \times\left\{\left[2-\frac{(1-\mu)}{2}k(k_1+1)\nabla^2-(1+\mu)k(k_1\frac{\partial^2}{\partial y^2}+\frac{\partial^2}{\partial x^2})\right]\phi-2\nabla^2 w\right\}=k(k_1-1)\left[(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}) -\frac{(1-\mu)}{2}k(k_1\frac{\partial^2}{\partial x^2}-\frac{\partial^2}{\partial y^2})\nabla^2 -(1+\mu)k(k_1-1)\frac{\partial^4}{\partial x^2\partial y^2}\right]\phi \tag{2}$$

$$(D_1+D_2)\nabla^4 w-\left[\frac{1}{R_1}+\frac{\partial^2 w}{\partial x^2}\right]\frac{\partial^2 F}{\partial y^2} -\left[\frac{1}{R_2}+\frac{\partial^2 w}{\partial y^2}\right]\frac{\partial^2 F}{\partial x^2}+2\frac{\partial^2 w}{\partial x \partial y}\frac{\partial^2 F}{\partial x \partial y} +\bar{k}\nabla^2\phi-q=0 \tag{3}$$

where

$$k_1=G_{xz}/G_{yz} \tag{4}$$

G_{xz} and G_{yz} are the shear moduli of the core in the x and y directions, respectively, and the rest of the notations are defined in Ref. 1. It can be readily shown that when $k=1$, i.e., for the case of isotropic core, Eqs. (1-3) reduce to Fulton's equations [Ref. 1, Eqs. (26,30, and 32)] as they should be.

III. Example

As an example, the equations derived herein are used to obtain the critical load and the snap-through load of a simply supported square shallow barrel shell panel with orthotropic core (Fig. 1). In this case, $a=b$, $R_1=\infty$, $R_2=R$, and N^* is the

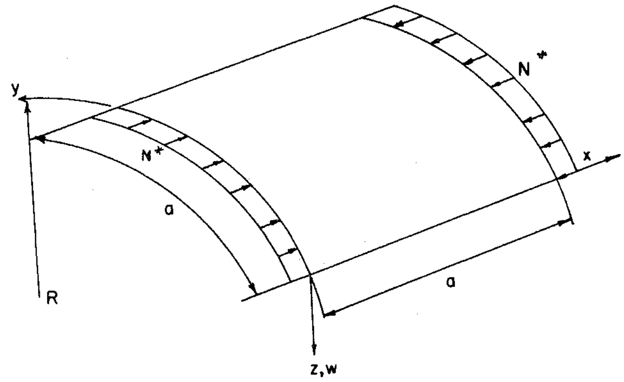

Fig. 1 Barrell shell subject to uniaxial loading.

normal load along the edges $x=o$ and $x=a$. Based on the given shell geometry, Eqs. (1-3) reduce to the following equations:

$$\nabla^4 F=(1-\mu^2)\,(B_1+B_2)\left[\left[\frac{\partial^2 w}{\partial x\partial y}\right]^2 - \frac{\partial^2 w}{\partial x^2}\,\frac{\partial^2 w}{\partial y^2} - \frac{1}{R}\,\frac{\partial^2 w}{\partial x^2}\right] \qquad (5)$$

$$\left[k\left[\frac{\partial^2}{\partial y^2}+k_1\frac{\partial^2}{\partial x^2}\right]-\frac{2}{(1-\mu)}\right]$$
$$\times\left\{\left[2-\frac{(1-\mu)}{2}k(k_1+1)\nabla^2-(1+\mu)\right.\right.$$
$$\left.\left.\times k\left(k_1\frac{\partial^2}{\partial y^2}+\frac{\partial^2}{\partial x^2}\right)\right]\phi-2\nabla^2 w\right\}$$
$$=k(k_1-1)\left[\left[\frac{\partial^2}{\partial x^2}-\frac{\partial^2}{\partial y^2}\right]\right.$$
$$-\frac{(1-\mu)}{2}\,k\,\left(k_1\frac{\partial^2}{\partial x^2}-\frac{\partial^2}{\partial y^2}\right)\nabla^2$$
$$\left.-(1+\mu)\,k\,(k_1-1)\frac{\partial^4}{\partial x^2\partial y^2}\right]\phi \qquad (6)$$

$$(D_1+D_2)\nabla^4 w-\frac{\partial^2 w}{\partial x^2}\,\frac{\partial^2 F}{\partial y^2}-\left(\frac{1}{R}+\frac{\partial^2 w}{\partial y^2}\right)\frac{\partial^2 F}{\partial x^2}$$
$$+2\,\frac{\partial^2 w}{\partial x\partial y}\,\frac{\partial^2 F}{\partial x\partial y}+\bar{k}\nabla^2\phi=0 \qquad (7)$$

The appropriate boundary conditions for this problem are[1]: Along the boundaries $x=o$ and $x=a$,

$$\partial^2F/\partial y^2=o,\ \partial^2F/\partial x\partial y=o,\ \phi=o,\ w=o,\ M^*_x=o \qquad (8)$$

Along the boundaries $y=o$ and $y=a$,

$$\partial^2F/\partial x^2=o,\ \partial^2F/\partial x\partial y=o,\ \phi=o,\ w=o,\ M^*_y=o \qquad (9)$$

The same method of solution presented in Ref. 1 will be used here. Assume that

$$w=w_o\sin\pi x/a\,\sin\pi y/a \qquad (10)$$

where w_o is a constant. Substituting Eq. (10) into Eq. (5) followed by integration yields

$$F=(C_1W_o^2/32)\,[\cos\,(2\pi x/a)+\cos\,(2\pi y/a)]$$
$$+C_1/4R(a/\pi)^2 w_o\sin\pi x/a\,\sin(\pi y/a)-\tfrac{1}{2}N^*y^2 \qquad (11)$$

Introducing Eqs. (10) and (11) in Eq. (2) and integrating yields

$$\phi=-2(\pi/a)^2 w_o\sin(\pi x/a)\,\sin(\pi y/a)$$
$$\times[2+(1-\mu)k(\pi/a)^2(k_1+1)]/$$
$$\{2[1+k(\pi/a)^2(k_1+1)]$$
$$+(1-\mu)k(\pi/a)^2[(k_1+1)+4kk_1(\pi/a)^2]\} \qquad (12)$$

Using these expressions for F and ϕ, Eq. (3) is then solved approximately using the method of Galerkin. After a lengthy process of integration, the following equation is obtained.[2]

$$-\frac{\pi^2}{4}N^*w_o+\frac{C_1\pi^4}{32a^2}w^3{}_o-\frac{C_1(1-\mu)}{4R}w_o^3$$
$$+(D_1+D_2)\frac{\pi^4}{a^2}w_o+\frac{C_1a^2}{16R^2}w_o$$
$$+\bar{k}\frac{\pi^4}{a^2}w_o[2+(1-\mu)k\frac{\pi^2}{a^2}(k_1+1)]/$$
$$\{2[1+k\frac{\pi^2}{a^2}(k_1+1)]$$
$$+(1-\mu)k\frac{\pi^2}{a^2}[(k_1+1)+4\,k_1k\frac{\pi^2}{a^2}]\}=0 \qquad (13)$$

Solving for N^* from Eq. (13) yields

$$N^*=4(D_1+D_2)\frac{\pi^2}{a^2}+\frac{C_1}{4R^2}\frac{a^2}{\pi^2}$$
$$+4\bar{k}\frac{\pi^2}{a^2}[2+(1-\mu)k\frac{\pi^2}{a^2}(k_1+1)]/$$
$$\{2[1+k\frac{\pi^2}{a^2}(k_1+1)]$$
$$+(1-\mu)k\frac{\pi^2}{a^2}[(k_1+1)+4\,k\frac{\pi^2}{a^2}k_1]\}$$
$$+\frac{C_1}{8}\frac{\pi^2}{a^2}w^2{}_o-\frac{C_1(1-\mu)}{\pi^2R}w_o \qquad (14)$$

The upper critical load N^*_u, is derived by dropping all the nonlinear terms in Eq. (13) and solving for N^*. Thus

$$N^*_u=4(D_1+D_2)\frac{\pi^2}{a^2}+\frac{C_1}{4R^2}\frac{a^2}{\pi^2}$$
$$+4\bar{k}\frac{\pi^2}{a^2}[2+(1-\mu)k\frac{\pi^2}{a^2}(k_1+1)]/$$
$$\{2[1+k\frac{\pi^2}{a^2}(k_1+1)]$$
$$+(1-\mu)k\frac{\pi^2}{a^2}[(k_1+1)+4\,k\frac{\pi^2}{a^2}k_1]\} \qquad (15)$$

The lower critical load N^*_L, which is corresponding to the load for the snap-through condition, is obtained by including the nonlinear terms in Eq. (13). Differentiating Eq. (14) with respect to w_o and solving for w_o from the resulting equation yields

$$w_o=(4a^2/\pi^4R)\;(1-\mu) \qquad (16)$$

The lower critical load N^*_L, then takes the form

$$N^*_L=N^*_u-[2C_1(1-\mu)^2/\pi^6]\;a^2/R^2 \qquad (17)$$

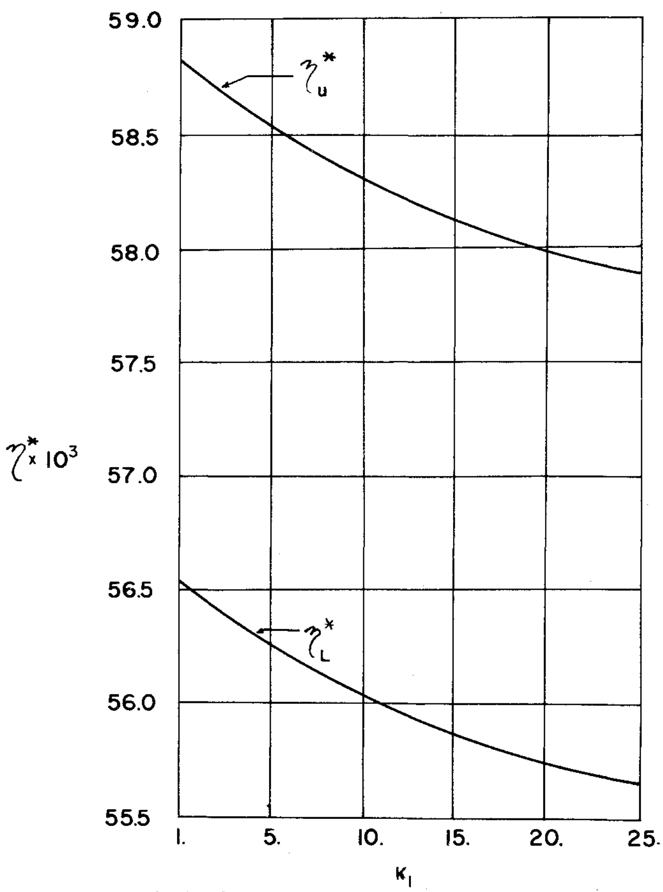

Fig. 2 Coefficients for uniaxial buckling.

For $k=1$, i.e., for the case of isotropic core, the critical loads N_u^* and N_L^*, given by Eqs. (16) and (17) reduce to those obtained by Fulton.[1]

In order to show the effect of orthotropic core upon the critical loads, the following numerical values for shell property and dimensions are used: $E_1=E_2=E$, $t_1=t_2=t=0.05$ in., $\mu=0.3$, $E/G_{xz}=2\times10^3$, $a=100$ in., $R=200$ in., $C=0.5$ in. The critical loads are expressed in the following form as

$$N_u^* = \eta_u^* (\pi^2 D/a^2) \quad (18)$$

$$N_L^* = \eta_L^* (\pi^2 D/a^2) \quad (19)$$

where η_u^* and η_L^* are the load coefficients. The values assumed for k_l ranged from 1-25 at intervals of 5. The curves are shown in Fig. 2. It can be seen from Fig. 2 that as k_l increases, the values for η_u^* and η_L^* decrease. The η^*-values show a maximum average of 0.42% decrease in the first k_l interval and the average rate of 0.29% decrease in the last k_l interval.

IV. Discussion and Conclusions

The nonlinear differential equations for shallow sandwich shells with orthotropic cores are derived herein. An example illustrating the use of these equations is presented. The approximate solution used in solving the example problem satisfies the boundary conditions for w, ϕ, M_x, and M_y given in Eqs. (8) and (9); however, the boundary conditions on F given in Eqs. (8) and (9) are satisfied only on the average. It should be noted that Fig. 2 represents load coefficients for a particular shell property and dimensions. The assumed deflection shape given by Eq. (10) for a square shallow shell represents the buckling mode shape of a half sine wave in both x and y directions. For a more general case of shell dimensions and property, the deflection function capable of representing any number of half-waves in both x and y directions should be used instead of Eq. (10), and the critical loads should be obtained from Eqs. (15) and (17).

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Convergence and Stability of Nonlinear Finite Element Equations

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IN recent years, the field of computational mechanics has broadened significantly, particularly with the finite element analysis applied to solution of initial and/or boundary value problems in both solids and fluids. The question is, "Are we assured of convergence and stability in the solution?" In general, the finite element equations occur in the form

$$A_{ij}(u_j)u_j - f_i = 0 \quad (1)$$

for the steady state, and

$$B_{ij}\dot{u}_j + C_{ij}(u_j)u_j - f_i = 0 \quad (2)$$

for the unsteady state. Here the superposed dot denotes a time derivative.

We concern ourselves with the convergence criteria of Eq. (1) and both stability and convergence criteria of Eq. (2) in the solutions of these equations. The literature on the subject of convergence and stability is abundant; for example, Ortega and Rheinboldt[1] for nonlinear equations and Richtmyer and Morton[2] for time-dependent equations. Their discussions are concerned with approximate numerical solutions via finite difference equations. Subsequently, Oden,[3] Fujii,[4] and others studied the problems of convergence and stability associated with finite elements. The present study is intended for derivations of explicit convergence criteria for nonlinear finite element equations and stability criteria for linear and nonlinear time dependent finite element equations.

Consider the nonlinear finite element equations of the form

$$R_i(u_j) = A_{ij}(u_j)u_j - f_i = 0 \quad (3)$$

Expanding Eq. (3) in Taylor series and retaining only the first-order terms yield

$$R_i(u_j) = R_i(u_j + \Delta u_j) = R_i(u_j^\circ) + [\partial R_i(u_j^\circ)/\partial u_j]\ (u_j - u_j^\circ) = 0 \quad (4)$$

Solving for u_i in Eq. (4) gives

$$u_i = u_i^\circ - (J_{ij}^\circ)^{-1} R_j(u_j^\circ) \quad (5)$$

where J_{ij}° is the Jacobian defined as

$$J_{ij}^\circ = \partial R_i(u_j^\circ)/\partial u_j \quad (6)$$

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